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## Deteriorating Items Inventory Model for Defective Items under Different Deterioration with Three Tired Prices and Time Dependent Demand

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**ABSTRACT**: Lot received or units produced are not all perfect items. A time and price dependent demand inventory model is formulated when items produced or lot received is of defective quality. Three tired pricing is considered. For different situations, expression for total profit is derived to derive optimal solution. For parameter, post-optimality computations are also done.

**KEYWORDS**: Inventory model, Varying Deterioration, Price dependent demand, Time dependent demand, Three tire pricing, Defective items

#### I. INTRODUCTION

It is assumed that items can be stock up for an indefinite period to meet future demand. But we come across certain items which either worsen or become superseded during time of stocking. So, if deterioration rate of item is not very small, it's impact cannot be overlooked during inventory system modeling. An inventory model for fashion goods items whose value deteriorates during stocking period was developed by Whitin [1957]. A fixed deterioration rate inventory model was formulated by Ghare and Schrader [1963]. Subsequently, model of Ghare and Schrader [1963] was expanded by Covert and Philip [1973] considering changeable deterioration rate. A Weibull rate decay inventory model obtained by Aggarwal and Goel [1984] under selling price dependent demand. A price dependent demand under freight discounts and order level inventory model was formulated by Mukhopadhyay et al. [2004]. A selling price with order quantity dependent model was considered by Teng and Chang [2005]. Manna and Chaudhuri [2006] constructed a stock level model for declining items under ramp type demand. A two parameters Weibull deterioration rate and price dependent demand selling price dependent inventory model. A stock level model under stock size and price fluctuating demand with varying deterioration was constructed by Patel and Sheikh [2015]. A non-instantaneous deteriorating items stock level model was formulated by Tsao et al. [2017] with trade credit situation.

It happens many times that all items received in a lot are not 100% perfect items. And we say these items as imperfect quality items. Imperfect quality items stock level models have been formulated keeping in view this fact. Imperfect quality items inventory model for finding optimal ordering policy was obtained by Lee and Rosenblatt [1985]. When production process is not perfect, an inventory model was formulated by Cheng [1991]. It was assumed that demand of item depends on unit production cost. Salameh and Jaber [2000] formulated an inventory model when received items are not 100% perfect. Imperfect units are separated after 100% screening and their selling will be done at discounted price. A defective items production inventory model was obtained by Goyal and Barron [2002]. Papachristos and Konstantaras [2006] constructed a no shortage defective items inventory model where imperfect quality items are withdrawnat the end of cycle. An imperfect quality items EPQ model subject to learning effects was formulated by Jaber et al. [2008]. Hsu and Hsu [2012] constructed an imperfect quality items stock model under shortages. Patel and Patel [2012] considered an imperfect quality items inventory model for defective items. An EPQ model with rework was developed to determine jointly number of deliveries, refill lot size and selling price by Taleizadeh et al. (2015). Shukla et al. [2016] obtained an imperfect quality items production inventory model of deteriorating nature. A stock level model for faulty and deteriorating items was constructed by Khanna et al. [2017] under trade credit policy and price related demand. Uthayakumar and Sekar [2017] obtained an inventory model under multiple production setups for defective items with salvage value. A deteriorating items stock level model for defective items under shortages with different deterioration rates was formulated by Naik and Patel [2017].



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A varying deterioration inventory model for defective items is developed. Three tire pricing policy is adopted. Demand of items is taken as function of price and time. Stock outs are not allowed. Model is justified with numerical example and post-optimality computations.

#### **II. ASSUMPTIONS AND NOTATIONS**

#### **NOTATIONS:**

Notations used in model are:

D(t,p<sub>i</sub>): Price and time dependent demand  $(a_i+b_it-\rho_ip_i, a_i>0, 0< b_i<1, \rho_i>0, i=1,2,3$  in different intervals)

- c : Unit cost of purchasing of item
- p<sub>i</sub> : Selling price per unit of item in different intervals in a cycle
- d : Imperfect articles (%)
- 1-d : % of perfect articles
- $\lambda$  : Rate of monitoring
- SR : Revenue from sales
- A : Per order cost of replenishment
- z : Screening cost of one item
- p<sub>d</sub> : Defective items price of selling per unit
- h(t) : Cost of holding per unit  $(x + yt, x > 0, 0 \le y \le 1)$
- $\mu_1$ : Period of screening
- T : Inventory cycle time
- I(t): At time t, inventory size
- Q : Quantity required in a cycle
- $\theta$  : rate of deterioration during  $t_1 \le t \le t_2, \ 0 < \theta < 1$
- $\theta$  t : Rate of deterioration during  $t_2 \le t \le T$ ,  $0 \le \theta \le 1$
- $\pi$  : Items per unit profit.

#### **ASSUMPTIONS:**

Model is based on assumptions:

- Demand of item is function of time and prices.
- Product has infinite and instantaneous replenishment rate.
- There is zero lead time.
- There are no shortages of items.
- During screening process, demand occurs but is less than screening rate ( $\lambda$ ) i.e.  $(a_i+b_it-\rho_ip_i) < \lambda$ .
- Deterioration of items and defective items are independent.
- In each cycle, no repairing or replacement of deteriorated items.
- Only one item is taken for analysis.
- Varying holding cost is considered.
- Screening machine takes very less time for inspection of items for verification means we say that screening rate  $(\lambda)$  is sufficiently large.

#### III. MATHEMATICAL MODEL AND ANALYSIS

Items of amount Q are received at the starting of cycle. Out of Q units d% of items are defective items. These units, at rate of  $\lambda$  per unit time as shown in figure below go through screening process during time 0 to  $\mu_1$ . Items which are found to be perfect are separated and demand occurred during 0 to  $\mu_1$  will be fulfilled from these perfect quality items. And defective units, are sold at end of cycle at reduced price as a single batch. One cycle time is divided as  $(0, t_1)$ ,  $(t_1, t_2)$  and  $(t_2, T)$ . In period  $(0, t_1)$  there is no deterioration and price is  $p_1$ , in period  $(t_1, t_2)$  deterioration rate is  $\theta$  and price is  $p_3$ , (where  $p_1 > p_2 > p_3$ ). At end of a cycle because of deterioration and demand, level of inventory reaches to zero.

Also here 
$$\mu_1 = \frac{Q}{\lambda}$$
 (1)

and we put restriction for defective percentage (d) as: 
$$d \le 1 - \frac{(a_i + b_i t - \rho_i p_i)}{\lambda}$$
.

During cycle time  $(0 \le t \le T)$  inventory size is as shown below:

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(2)



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$$\frac{dI(t)}{dt} = -(a_1 + b_1 t - \rho_1 p_1), \qquad 0 \le t \le t_1(3)$$

$$\frac{dI(t)}{dt} + \theta I(t) = -(a_2 + b_2 t - \rho_2 p_2), \qquad t_1 \le t \le t_2(4)$$

$$\frac{dI(t)}{dt} + \theta tI(t) = -(a_3 + b_3 t - \rho_3 p_3), \qquad t_2 \le t \le T(5)$$

conditions initially taken as: I( 0 ) = Q, I( $t_1$ ) = S<sub>1</sub>, I(T)= 0. Respective are solutions:

$$I(t) = Q - (a_{1}t - \rho_{1}p_{1}t + \frac{1}{2}b_{1}t^{2})$$

$$I(t) = \begin{bmatrix} a_{2}(t_{1}-t) - \rho_{2}p_{2}(t_{1}-t) + \frac{1}{2}a_{2}\theta(t_{1}^{2}-t^{2}) - \frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2}-t^{2}) + \frac{1}{2}b_{2}(t_{1}^{2}-t^{2}) \\ + \frac{1}{3}b_{2}\theta(t_{1}^{3}-t^{3}) - a_{2}\theta t(t_{1}-t) + \rho_{2}p_{2}\theta t(t_{1}-t) - \frac{1}{2}b_{2}\theta t(t_{1}^{2}-t^{2}) \end{bmatrix} + S_{1}[1 + \theta(t_{1}-t)]$$

$$(6)$$

$$I(t) = \begin{bmatrix} a_{2}(t_{1}-t) - \rho_{2}p_{2}(t_{1}-t) + \frac{1}{2}a_{2}\theta(t_{1}^{2}-t^{2}) - \frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2}-t^{2}) + \frac{1}{2}b_{2}(t_{1}^{2}-t^{2}) \\ + \frac{1}{3}b_{2}\theta(t_{1}^{3}-t^{3}) - a_{2}\theta t(t_{1}-t) + \rho_{2}p_{2}\theta t(t_{1}-t) - \frac{1}{2}b_{2}\theta t(t_{1}^{2}-t^{2}) \end{bmatrix} + S_{1}[1 + \theta(t_{1}-t)]$$

$$(7)$$

$$I(t) = \begin{bmatrix} a_3 (T - t) - \rho_3 p_3 (T - t) + \frac{1}{2} b_3 (T^2 - t^2) + \frac{1}{6} a_3 \theta (T^3 - t^3) - \frac{1}{6} \rho_3 p_3 \theta (T^3 - t^3) \\ + \frac{1}{8} b_3 \theta (T^4 - t^4) - \frac{1}{2} a_3 \theta t^2 (T - t) + \frac{1}{2} \rho_3 p_3 \theta t^2 (T - t) - \frac{1}{4} b_3 \theta t^2 (T^2 - t^2) \end{bmatrix}$$
(8)

(higher powers of  $\theta$  are not considered) dQ is defective items separated at time  $\mu_1$  after screening process. Therefore between  $\mu_1 \le t \le T$ , effective inventory is:

$$I_{1}(t) = Q(1 - d) - (a_{1}t - \rho_{1}p_{1}t + \frac{1}{2}b_{1}t^{2}).$$
Substituting t = t<sub>1</sub>, in (6) gives
(9)

$$Q = S_{1} + \left(a_{1}t_{1} - \rho_{1}p_{1}t_{1} + \frac{1}{2}b_{1}t_{1}^{2}\right).$$
(10)

Taking  $t = t_2$  in equations (7) and (8), we get

$$I(t_{2}) = \begin{bmatrix} a_{2}(t_{1}-t_{2})-\rho_{2}p_{2}(t_{1}-t_{2})+\frac{1}{2}a_{2}\theta(t_{1}^{2}-t_{2}^{2})-\frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2}-t_{2}^{2})+\frac{1}{2}b_{2}(t_{1}^{2}-t_{2}^{2})\\ +\frac{1}{3}b_{2}\theta(t_{1}^{3}-t_{2}^{3})-a_{2}\theta t_{2} (t_{1}-t_{2})+\rho_{2}p_{2}\theta t_{2} (t_{1}-t_{2})-\frac{1}{2}b_{2}\theta\mu_{2}(t_{1}^{2}-t_{2}^{2}) \end{bmatrix} +S_{1}\left[1+\theta(t_{1}-t_{2})\right] (11)$$

$$I(t_{2}) = \begin{bmatrix} a_{3}(T-t_{2})-\rho_{3}p_{3}(T-t_{2})+\frac{1}{2}b_{3}(T^{2}-t_{2}^{2})+\frac{1}{6}a_{3}\theta(T^{3}-t_{2}^{3})-\frac{1}{6}\rho_{3}p_{3}\theta(T^{3}-t_{2}^{3})\\ +\frac{1}{8}b_{3}\theta(T^{4}-t_{2}^{4})-\frac{1}{2}a_{3}\theta t_{2}^{2}(T-t_{2})+\frac{1}{2}\rho_{3}p_{3}\theta\mu_{2}^{2}(T-t_{2})-\frac{1}{4}b_{3}\theta t_{2}^{2}(T^{2}-t_{2}^{2}) \end{bmatrix}$$
(12)

So from equations (11) and (12), we get

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$$S_{1} = \frac{1}{\left[1 + \theta(t_{1} - t_{2})\right]} \begin{vmatrix} a_{3}(T - t_{2}) - \rho_{3}p_{3}(T - t_{2}) + \frac{1}{2}b_{3}(T^{2} - t_{2}^{2}) + \frac{1}{6}a_{3}\theta(T^{3} - t_{2}^{3}) - \frac{1}{6}\rho_{3}p_{3}\theta(T^{3} - t_{2}^{3}) \\ + \frac{1}{8}b_{3}\theta(T^{4} - t_{2}^{4}) - \frac{1}{2}a_{3}\theta t_{2}^{2}(T - t_{2}) + \frac{1}{2}\rho_{3}p_{3}\theta \mu_{2}^{2}(T - t_{2}) - \frac{1}{4}b_{3}\theta t_{2}^{2}(T^{2} - t_{2}^{2}) \\ - a_{2}(t_{1} - t_{2}) + \rho_{2}p_{2}(t_{1} - t_{2}) - \frac{1}{2}a_{2}\theta(t_{1}^{2} - t_{2}^{2}) + \frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2} - t_{2}^{2}) - \frac{1}{2}b_{2}(t_{1}^{2} - t_{2}^{2}) \\ - \frac{1}{3}b_{2}\theta(t_{1}^{3} - t_{2}^{3}) + a_{2}\theta t_{2}(t_{1} - t_{2}) - \rho_{2}p_{2}\theta t_{2}(t_{1} - t_{2}) + \frac{1}{2}b_{2}\theta t_{2}(t_{1}^{2} - t_{2}^{2}) \\ - \frac{1}{3}b_{2}\theta(t_{1}^{3} - t_{2}^{3}) + a_{2}\theta t_{2}(t_{1} - t_{2}) - \rho_{2}p_{2}\theta t_{2}(t_{1} - t_{2}) + \frac{1}{2}b_{2}\theta t_{2}(t_{1}^{2} - t_{2}^{2}) \\ \end{bmatrix}$$
We get equation (10) by substituting value of S<sub>1</sub> from equation (13)

get equation (10) by substituting value of  $S_1$  from equation (13)

$$Q = \frac{1}{\left[1 + \theta(t_{1} - t_{2})\right]} \begin{bmatrix} a_{3}(T - t_{2}) - \rho_{3}p_{3}(T - t_{2}) + \frac{1}{2}b_{3}(T^{2} - t_{2}^{2}) + \frac{1}{6}a_{3}\theta(T^{3} - t_{2}^{3}) - \frac{1}{6}\rho_{3}p_{3}\theta(T^{3} - t_{2}^{3}) \\ + \frac{1}{8}b_{3}\theta(T^{4} - t_{2}^{4}) - \frac{1}{2}a_{3}\theta t_{2}^{2}(T - t_{2}) + \frac{1}{2}\rho_{3}p_{3}\theta \mu_{2}^{2}(T - t_{2}) - \frac{1}{4}b_{3}\theta t_{2}^{2}(T^{2} - t_{2}^{2}) \\ - a_{2}(t_{1} - t_{2}) + \rho_{2}p_{2}(t_{1} - t_{2}) - \frac{1}{2}a_{2}\theta(t_{1}^{2} - t_{2}^{2}) + \frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2} - t_{2}^{2}) - \frac{1}{2}b_{2}(t_{1}^{2} - t_{2}^{2}) \\ - \frac{1}{3}b_{2}\theta(t_{1}^{3} - t_{2}^{3}) + a_{2}\theta t_{2}(t_{1} - t_{2}) - \rho_{2}p_{2}\theta t_{2}(t_{1} - t_{2}) + \frac{1}{2}b_{2}\theta t_{2}(t_{1}^{2} - t_{2}^{2}) \\ + \left(a_{1}t_{1} - \rho_{1}p_{1}t_{1} + \frac{1}{2}b_{1}t_{1}^{2}\right)$$
(14)

Using (14) in (6), we have

$$I_{1}(t) = \frac{1}{\left[1 + \theta(t_{1} - t_{2})\right]} \begin{vmatrix} a_{3}(T - t_{2}) - \rho_{3}p_{3}(T - t_{2}) + \frac{1}{2}b_{3}(T^{2} - t_{2}^{2}) + \frac{1}{6}a_{3}\theta(T^{3} - t_{2}^{3}) - \frac{1}{6}\rho_{3}p_{3}\theta(T^{3} - t_{2}^{3}) \\ + \frac{1}{8}b_{3}\theta(T^{4} - t_{2}^{4}) - \frac{1}{2}a_{3}\theta t_{2}^{2}(T - t_{2}) + \frac{1}{2}\rho_{3}p_{3}\theta \mu_{2}^{2}(T - t_{2}) - \frac{1}{4}b_{3}\theta t_{2}^{2}(T^{2} - t_{2}^{2}) \\ -a_{2}(t_{1} - t_{2}) + \rho_{2}p_{2}(t_{1} - t_{2}) - \frac{1}{2}a_{2}\theta(t_{1}^{2} - t_{2}^{2}) + \frac{1}{2}\rho_{2}p_{2}\theta(t_{1}^{2} - t_{2}^{2}) - \frac{1}{2}b_{2}(t_{1}^{2} - t_{2}^{2}) \\ -\frac{1}{3}b_{2}\theta(t_{1}^{3} - t_{2}^{3}) + a_{2}\theta t_{2}(t_{1} - t_{2}) - \rho_{2}p_{2}\theta t_{2}(t_{1} - t_{2}) + \frac{1}{2}b_{2}\theta t_{2}(t_{1}^{2} - t_{2}^{2}) \\ + \left(a_{1}t_{1} - \rho_{1}p_{1}t_{1} + \frac{1}{2}b_{1}t_{1}^{2}\right) - \left(a_{1}t - \rho_{1}p_{1}t + \frac{1}{2}b_{1}t^{2}\right)$$
(15)

Similarly, using (14) in (9), we have

$$I_{1}(t) = \frac{(1-d)}{\left[1+\theta\left(t_{1}-t_{2}\right)\right]} \left[ \begin{array}{c} a_{3}\left(T-t_{2}\right)+\frac{1}{2}b_{3}\left(T^{2}-t_{2}^{2}\right)+\frac{1}{6}a_{3}\theta\left(T^{3}-t_{2}^{3}\right)-a_{2}\left(t_{1}-t_{2}\right)\right) \\ -\frac{1}{6}\rho_{3}p_{3}\theta\left(T^{3}-t_{2}^{3}\right)+\frac{1}{8}b_{3}\theta\left(T^{4}-t_{2}^{4}\right)-\frac{1}{2}a_{3}\theta t_{2}^{2}\left(T-t_{2}\right) \\ +\frac{1}{2}\rho_{3}p_{3}\theta\mu_{2}^{2}\left(T-t_{2}\right)-\frac{1}{4}b_{3}\theta t_{2}^{2}\left(T^{2}-t_{2}^{2}\right)+\frac{1}{2}b_{2}\theta t_{2}\left(t_{1}^{2}-t_{2}^{2}\right) \\ +\rho_{2}p_{2}\left(t_{1}-t_{2}\right)-\frac{1}{2}a_{2}\theta\left(t_{1}^{2}-t_{2}^{2}\right)+\frac{1}{2}\rho_{2}p_{2}\theta\left(t_{1}^{2}-t_{2}^{2}\right)-\frac{1}{2}b_{2}\left(t_{1}^{2}-t_{2}^{2}\right) \\ -\frac{1}{3}b_{2}\theta\left(t_{1}^{3}-t_{2}^{3}\right)+a_{2}\theta t_{2}\left(t_{1}-t_{2}\right)-\rho_{2}p_{2}\theta t_{2}\left(t_{1}-t_{2}\right) \\ +\left(1-d\right)\left(a_{1}t_{1}-\rho_{1}p_{1}t_{1}+\frac{1}{2}b_{1}t_{1}^{2}\right)-\left(a_{1}t-\rho_{1}p_{1}t+\frac{1}{2}b_{1}t^{2}\right).$$

$$(16)$$

We get equation (7) by substituting value of  $S_1$  from equation (13)



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$$\begin{split} \mathbf{I}_{2}(t) &= \frac{\left[1\!+\!\theta(t_{1}\!-\!t)\right]}{\left[1\!+\!\theta(t_{1}\!-\!t_{2})\right]} \begin{bmatrix} a_{3}\left(T\!-\!t_{2}\right)\!-\!\rho_{3}p_{3}\left(T\!-\!t_{2}\right)\!+\!\frac{1}{2}b_{3}\left(T^{2}\!-\!t_{2}^{2}\right)\!+\!\frac{1}{6}a_{3}\theta\left(T^{3}\!-\!t_{2}^{3}\right)\!-\!\frac{1}{6}\rho_{3}p_{3}\theta\left(T^{3}\!-\!t_{2}^{3}\right) \\ &+\frac{1}{8}b_{3}\theta\left(T^{4}\!-\!t_{2}^{4}\right)\!-\!\frac{1}{2}a_{3}\theta t_{2}^{2}\left(T\!-\!t_{2}\right) + \frac{1}{2}\rho_{3}p_{3}\theta \mu_{2}^{2}\left(T\!-\!t_{2}\right)\!-\!\frac{1}{4}b_{3}\theta t_{2}^{2}\left(T^{2}\!-\!t_{2}^{2}\right) \\ &-a_{2}\left(t_{1}\!-\!t_{2}\right)\!+\!\rho_{2}p_{2}\left(t_{1}\!-\!t_{2}\right)\!-\!\frac{1}{2}a_{2}\theta\left(t_{1}^{2}\!-\!t_{2}^{2}\right)\!+\!\frac{1}{2}\rho_{2}p_{2}\theta\left(t_{1}^{2}\!-\!t_{2}^{2}\right)\!-\!\frac{1}{2}b_{2}\left(t_{1}^{2}\!-\!t_{2}^{2}\right) \\ &-\frac{1}{3}b_{2}\theta\left(t_{1}^{3}\!-\!t_{2}^{3}\right)\!+a_{2}\theta t_{2}\left(t_{1}\!-\!t_{2}\right)\!-\!\rho_{2}p_{2}\theta t_{2}\left(t_{1}\!-\!t_{2}\right)\!+\!\frac{1}{2}b_{2}\theta t_{2}\left(t_{1}^{2}\!-\!t_{2}^{2}\right) \\ &+\left[a_{2}\left(t_{1}\!-\!t\right)\!-\!\rho_{2}p_{2}\left(t_{1}\!-\!t\right)\!+\!\frac{1}{2}a_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!-\!\frac{1}{2}\rho_{2}p_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!+\!\frac{1}{2}b_{2}\left(t_{1}^{2}\!-\!t^{2}\right) \\ &+\left[a_{2}\left(t_{1}\!-\!t\right)\!-\!\rho_{2}p_{2}\left(t_{1}\!-\!t\right)\!+\!\frac{1}{2}a_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!-\!\frac{1}{2}\rho_{2}p_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!+\!\frac{1}{2}b_{2}\left(t_{1}^{2}\!-\!t^{2}\right) \\ &+\left[a_{2}\left(t_{1}\!-\!t\right)\!-\!\rho_{2}p_{2}\left(t_{1}\!-\!t\right)\!+\!\frac{1}{2}a_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!-\!\frac{1}{2}\rho_{2}p_{2}\theta\left(t_{1}^{2}\!-\!t^{2}\right)\!+\!\frac{1}{2}b_{2}\left(t_{1}^{2}\!-\!t^{2}\right)\right]\right].$$
(17)

Total profit ( $\pi$ ) include:

(iii) HC = 
$$\int_{0}^{t} (x + yt) I(t) dt$$
  
=  $\int_{0}^{\mu_{1}} (x + yt) I_{1}(t) dt + \int_{\mu_{1}}^{t_{1}} (x + yt) I_{1}(t) dt + \int_{t_{1}}^{t_{2}} (x + yt) I_{2}(t) dt + \int_{t_{2}}^{T} (x + yt) I_{3}(t) dt$  (20)  
(iv) DC = c  $\left(\int_{t_{1}}^{t_{2}} \theta I_{2}(t) dt + \int_{t_{2}}^{T} \theta t I_{3}(t) dt\right)$  (21)

(v) 
$$SR = \left( p_1 \int_{0}^{t_1} (a_1 + b_1 t - \rho_1 p_1) dt + p_2 \int_{t_1}^{t_2} (a_2 + b_2 t - \rho_2 p_2) dt + p_3 \int_{t_2}^{T} (a_3 + b_3 t - \rho_3 p_3) dt + (p_d - p_1) dQ \right).$$
 (22)  
(higher powers of  $\theta$  are not considered)

Total profit

$$\pi = \frac{1}{T} \left[ SR - OC - SrC - HC - DC \right]$$
(23)

Putting value in equation (23) from equations (18) to (22) provides overall unit profit. Taking  $t_1 = v_1 T$ ,  $t_2 = v_2 T$  in equation (23), profit in terms of p1, p2, p3 and T can be obtained. Taking derivative with respect to p1, p2, p3, T and equating it to zero, in equation (23), gives

i.e. 
$$\frac{\partial \pi}{\partial p_1} = 0$$
,  $\frac{\partial \pi}{\partial p_2} = 0$ ,  $\frac{\partial \pi}{\partial p_3} = 0$ ,  $\frac{\partial \pi}{\partial T} = 0$ , (24)  
under fulfilling of the situation

$$\begin{vmatrix} \frac{\partial \pi^2}{\partial \mathbf{p}_1^2} & \frac{\partial \pi^2}{\partial \mathbf{p}_1 \partial \mathbf{p}_2} & \frac{\partial \pi^2}{\partial \mathbf{p}_1 \partial \mathbf{p}_3} & \frac{\partial \pi^2}{\partial \mathbf{p}_1 \partial \mathbf{T}} \\ \frac{\partial \pi^2}{\partial \mathbf{p}_2 \partial \mathbf{p}_1} & \frac{\partial \pi^2}{\partial^2 \mathbf{p}_2^2} & \frac{\partial \pi^2}{\partial \mathbf{p}_2 \partial \mathbf{p}_3} & \frac{\partial \pi^2}{\partial \mathbf{p}_2 \partial \mathbf{T}} \\ \frac{\partial \pi^2}{\partial \mathbf{p}_3 \partial \mathbf{p}_1} & \frac{\partial \pi^2}{\partial \mathbf{p}_3 \partial \mathbf{p}_2} & \frac{\partial \pi^2}{\partial^2 \mathbf{p}_3^2} & \frac{\partial \pi^2}{\partial \mathbf{p}_3 \partial \mathbf{T}} \\ \frac{\partial \pi^2}{\partial \mathbf{T} \partial \mathbf{p}_1} & \frac{\partial \pi^2}{\partial \mathbf{T} \partial \mathbf{p}_2} & \frac{\partial \pi^2}{\partial \mathbf{T} \partial \mathbf{p}_3} & \frac{\partial \pi^2}{\partial^2 \mathbf{T}^2} \end{vmatrix} > 0.$$

(25)

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#### **IV. NUMERICAL EXAMPLE**

Considering A = 100,  $a_1 = 500$ ,  $a_2 = 490$ ,  $a_3 = 480$ ,  $b_1 = 0.04$ ,  $b_2 = 0.05$ ,  $b_3 = 0.06$ , c = 25,  $p_d = 15$ , d = 0.02, z=0.40,  $\theta = 0.05, x = 5, y = 0.05, \rho_1 = 4, \rho_2 = 4.5, \rho_3 = 5, v_1 = 0.30, v_2 = 0.50$ , in proper units. Optimal values are:  $p_1^* = 61.2934, p_2^*$ = 55.5242, p<sub>3</sub>\*=49.4711, T\* = 0.3940, Profit\* = Rs. 12285.3548 and Q\*=95.1572. Equation (25) is also satisfied. Graphs for prices and profit are also shown below.



#### V. POST-OPTIMALITY COMPUTATIONS

Study of one parameter at a time, table below gives post-optimality computations.

Post-optimality Analysis										
Parameter	%	Т	<b>p</b> <sub>1</sub>	$p_2$	<b>p</b> <sub>3</sub>	Profit	Q			
a <sub>1</sub>	+20%	0.3910	73.7959	55.6465	49.5906	14259.6288	100.1333			
	+10%	0.3925	67.5447	55.5853	49.5308	13226.5375	97.6562			
	-10%	0.3955	55.0422	55.4632	49.4113	11436.0809	92.6363			
	-20%	0.3970	48.7909	55.4021	49.3516	10678.7156	90.0935			
a <sub>2</sub>	+20%	0.3878	61.2075	66.4058	49.4573	13438.1188	97.5183			
	+10%	0.3909	61.2504	60.9650	49.4641	12835.0412	96.3533			
	-10%	0.3971	61.3364	50.0836	49.4782	11789.0605	93.9300			
	-20%	0.4004	61.3795	44.6430	49.4854	11346.1590	92.7182			
a <sub>3</sub>	+20%	0.3667	61.0773	55.4933	59.0117	14750.7824	97.5451			
	+10%	0.3796	61.1851	55.5080	54.2398	13460.1146	96.3301			
	-10%	0.4101	61.4026	55.5425	44.7062	11226.5875	94.0152			
	-20%	0.4286	61.5127	55.5630	39.9462	10283.9158	92.9917			
x	+20%	0.3631	61.3112	55.5635	49.5420	12241.2026	87.5846			
	+10%	0.3776	61.3025	55.5443	49.5073	12262.8204	91.1374			
	-10%	0.4126	61.2838	55.5031	49.4332	12308.9302	99.7184			
	-20%	0.4342	61.2735	55.4808	49.3935	12333.7011	105.0175			
θ	+20%	0.3904	61.2911	55.5257	49.4777	12281.5262	94.3335			
	+10%	0.3922	61.2923	55.5250	49.4744	12283.4355	94.7456			
	-10%	0.3958	61.2946	55.5235	49.4678	12287.2846	95.5683			
	-20%	0.3977	61.2958	55.5228	49.4644	12289.2248	96.0031			
	+20%	0.4310	61.3084	55.5637	49.5494	12236.8721	104.0212			
	+10%	0.4129	61.3011	55.5444	49.5111	12260.5696	99.6864			

Table 1								
Post-optimality Ana								
-	<b>n</b>							

1



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А	-10%	0.3740	61.2853	55.5030	49.4290	12311.3958	90.3608
	-20%	0.3529	61.2768	55.4805	49.3846	12338.9087	85.2970
	+20%	0.3937	51.1081	55.4219	49.3682	11553.7704	95.2055
	+10%	0.3938	55.7377	55.4684	49.4149	11886.2995	95.1756
$\rho_1$	-10%	0.3941	68.0842	55.5925	49.5397	12773.1190	95.0976
	-20%	0.3943	76.5733	55.6777	49.6255	13382.8699	95.0388
	+20%	0.3941	61.2951	46.4502	49.4713	11840.9205	95.1037
	+10%	0.3940	61.2943	50.5747	49.4712	12042.9266	95.1183
$\rho_2$	-10%	0.3939	61.2926	61.5737	49.4709	12581.6794	95.1719
	-20%	0.3939	61.2917	69.1354	49.4708	12952.1112	95.2107
	+20%	0.3948	61.3001	55.5252	41.4727	11326.3647	95.0537
	+10%	0.3944	61.2967	55.5247	45.1082	11762.2198	95.1058
ρ <sub>3</sub>	-10%	0.3935	61.2901	55.5238	54.8037	12924.8627	95.1838
	-20%	0.3931	61.2868	55.5233	61.4697	13724.3830	95.2342

Calculations of Table 1 shows that increase or decrease in value of profit and order quantity occur, when parameters ' $a_1$ ', ' $a_2$ ', ' $a_3$ ' increase/decrease.

Also when parameters ' $\theta$ ' and 'x' increase/ decrease then total profit and order quantity decrease/ increase.

Moreover, when increase/decrease in parameters 'A' and ' $\rho_1$ ' occurs, then profit also shows decrease/increase and order quantity shows increase/ decrease.

Also when parameters ' $\rho_2$ ' and ' $\rho_3$ ' increase/ decrease then profit and order quantity decrease/ increase.

#### VI. SPECIAL CASE

Taking d=0, p<sub>d</sub>=0, z=0 gives

 $T^* = 0.3908$ ,  $p_1^* = 62.6469$ ,  $p_2^* = 54.8515$ ,  $p_3^* = 48.8029$ , Profit\*= Rs. 12606.6509. The result corresponds with result obtained by Patel and Patel [2021].

#### VII. CONCLUSION

Price and time dependent demand for imperfect quality items under three tire pricing inventory model is developed. For major parameters, post-optimality analysis is done. There will be variations in profit and order quantity with variations in parameter values.

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